Archetype Summary

to accompany

A First Course in Linear Algebra

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Type	S	S	S	S	S	S	S	S	S	S	M	M	L	L	L	L	L	L	L	L	L	L	L	L
Vars, Cols, Domain	3	3	4	4	4	4	2	2	7	9	5	5	5	5	3	3	5	5	3	5	6	4	3	4
Eqns, Rows, CoDom	3	3	3	3	3	4	5	5	4	6	5	5	3	3	5	5	5	5	4	6	4	4	3	4
Solution Set	I	U	I	I	N	U	U	N	I	I														
Rank	2	3	3	2	2	4	2	2	3	4	5	3	2	3	2	3	4	5	2	5	4	4	3	3
Nullity	1	0	1	2	2	0	0	0	4	5	0	2	3	2	1	0	1	0	1	0	2	0	0	1
Injective													X	X	N	Y	N	Y	N	Y	X	Y	Y	N
Surjective													N	Y	X	X	N	Y	X	X	Y	Y	Y	N
Full Rank	N	Y	Y	N	N	Y	Y	Y	N	N	Y	N												
Nonsingular	N	Y				Y					Y	N												
Invertible	N	Y				Y					Y	N					N	Y				Y	Y	N
Determinant	0	-2				-18					16	0										-2	-3	0
Diagonalizable	N	Y				Y					Y	Y											Y	Y

Archetype Facts

S=System of Equations, M=Matrix, L=Linear Transformation
U=Unique solution, I=Infinitely many solutions, N=No solutions
Y=Yes, N=No, X=Impossible, blank=Not Applicable

Archetype A

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 8 \\ x_1 + x_2 &= 5 \end{aligned}$$

Archetype B

$$\begin{aligned} -7x_1 - 6x_2 - 12x_3 &= -33 \\ 5x_1 + 5x_2 + 7x_3 &= 24 \\ x_1 + 4x_3 &= 5 \end{aligned}$$

Archetype C

$$\begin{aligned} 2x_1 - 3x_2 + x_3 - 6x_4 &= -7 \\ 4x_1 + x_2 + 2x_3 + 9x_4 &= -7 \\ 3x_1 + x_2 + x_3 + 8x_4 &= -8 \end{aligned}$$

Archetype D

$$\begin{aligned} 2x_1 + x_2 + 7x_3 - 7x_4 &= 8 \\ -3x_1 + 4x_2 - 5x_3 - 6x_4 &= -12 \\ x_1 + x_2 + 4x_3 - 5x_4 &= 4 \end{aligned}$$

Archetype E

$$\begin{aligned} 2x_1 + x_2 + 7x_3 - 7x_4 &= 2 \\ -3x_1 + 4x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + x_2 + 4x_3 - 5x_4 &= 2 \end{aligned}$$

Archetype F

$$\begin{aligned} 33x_1 - 16x_2 + 10x_3 - 2x_4 &= -27 \\ 99x_1 - 47x_2 + 27x_3 - 7x_4 &= -77 \\ 78x_1 - 36x_2 + 17x_3 - 6x_4 &= -52 \\ -9x_1 + 2x_2 + 3x_3 + 4x_4 &= 5 \end{aligned}$$

Archetype G

$$\begin{aligned}
2x_1 + 3x_2 &= 6\\
-x_1 + 4x_2 &= -14\\
3x_1 + 10x_2 &= -2\\
3x_1 - x_2 &= 20\\
6x_1 + 9x_2 &= 18
\end{aligned}$$

Archetype H

$$\begin{aligned}
2x_1 + 3x_2 &= 5\\
-x_1 + 4x_2 &= 6\\
3x_1 + 10x_2 &= 2\\
3x_1 - x_2 &= -1\\
6x_1 + 9x_2 &= 3
\end{aligned}$$

Archetype I

$$\begin{aligned}
x_1 + 4x_2 - x_4 + 7x_6 - 9x_7 &= 3\\
2x_1 + 8x_2 - x_3 + 3x_4 + 9x_5 - 13x_6 + 7x_7 &= 9\\
2x_3 - 3x_4 - 4x_5 + 12x_6 - 8x_7 &= 1\\
-x_1 - 4x_2 + 2x_3 + 4x_4 + 8x_5 - 31x_6 + 37x_7 &= 4
\end{aligned}$$

Archetype J

$$\begin{aligned}
x_1 + 2x_2 - 2x_3 + 9x_4 + 3x_5 - 5x_6 - 2x_7 + x_8 + 27x_9 &= -5\\
2x_1 + 4x_2 + 3x_3 + 4x_4 - x_5 + 4x_6 + 10x_7 + 2x_8 - 23x_9 &= 18\\
x_1 + 2x_2 + x_3 + 3x_4 + x_5 + x_6 + 5x_7 + 2x_8 - 7x_9 &= 6\\
2x_1 + 4x_2 + 3x_3 + 4x_4 - 7x_5 + 2x_6 + 4x_7 - 11x_9 &= 20\\
x_1 + 2x_2 + 5x_4 + 2x_5 - 4x_6 + 3x_7 + 8x_8 + 13x_9 &= -4\\
-3x_1 - 6x_2 - x_3 - 13x_4 + 2x_5 - 5x_6 - 4x_7 + 13x_8 + 10x_9 &= -29
\end{aligned}$$

Archetype K

$$\begin{bmatrix} 10 & 18 & 24 & 24 & -12 \\ 12 & -2 & -6 & 0 & -18 \\ -30 & -21 & -23 & -30 & 39 \\ 27 & 30 & 36 & 37 & -30 \\ 18 & 24 & 30 & 30 & -20 \end{bmatrix}$$

Archetype L

$$\begin{bmatrix} -2 & -1 & -2 & -4 & 4 \\ -6 & -5 & -4 & -4 & 6 \\ 10 & 7 & 7 & 10 & -13 \\ -7 & -5 & -6 & -9 & 10 \\ -4 & -3 & -4 & -6 & 6 \end{bmatrix}$$

Archetype M

$$T\colon \mathbb{C}^5 \mapsto \mathbb{C}^3, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 + 3x_3 + 4x_4 + 4x_5 \\ 3x_1 + x_2 + 4x_3 - 3x_4 + 7x_5 \\ x_1 - x_2 - 5x_4 + x_5 \end{bmatrix}$$

Archetype N

$$T\colon \mathbb{C}^5 \mapsto \mathbb{C}^3, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_2 + 3x_3 - 4x_4 + 5x_5 \\ x_1 - 2x_2 + 3x_3 - 9x_4 + 3x_5 \\ 3x_1 + 4x_3 - 6x_4 + 5x_5 \end{bmatrix}$$

Archetype O

$$T\colon \mathbb{C}^3 \mapsto \mathbb{C}^5, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + x_2 - 3x_3 \\ -x_1 + 2x_2 - 4x_3 \\ x_1 + x_2 + x_3 \\ 2x_1 + 3x_2 + x_3 \\ x_1 + 2x_3 \end{bmatrix}$$

Archetype P

$$T\colon \mathbb{C}^3 \mapsto \mathbb{C}^5, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + x_2 + x_3 \\ -x_1 + 2x_2 + 2x_3 \\ x_1 + x_2 + 3x_3 \\ 2x_1 + 3x_2 + x_3 \\ -2x_1 + x_2 + 3x_3 \end{bmatrix}$$

Archetype Q

$$T\colon \mathbb{C}^5 \mapsto \mathbb{C}^5, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right) = \begin{bmatrix} -2x_1 + 3x_2 + 3x_3 - 6x_4 + 3x_5 \\ -16x_1 + 9x_2 + 12x_3 - 28x_4 + 28x_5 \\ -19x_1 + 7x_2 + 14x_3 - 32x_4 + 37x_5 \\ -21x_1 + 9x_2 + 15x_3 - 35x_4 + 39x_5 \\ -9x_1 + 5x_2 + 7x_3 - 16x_4 + 16x_5 \end{bmatrix}$$

Archetype R

$$T\colon \mathbb{C}^5 \mapsto \mathbb{C}^5, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right) = \begin{bmatrix} -65x_1 + 128x_2 + 10x_3 - 262x_4 + 40x_5 \\ 36x_1 - 73x_2 - x_3 + 151x_4 - 16x_5 \\ -44x_1 + 88x_2 + 5x_3 - 180x_4 + 24x_5 \\ 34x_1 - 68x_2 - 3x_3 + 140x_4 - 18x_5 \\ 12x_1 - 24x_2 - x_3 + 49x_4 - 5x_5 \end{bmatrix}$$

Archetype S

$$T\colon \mathbb{C}^3 \mapsto M_{22}, \quad T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a - b & 2a + 2b + c \\ 3a + b + c & -2a - 6b - 2c \end{bmatrix}$$

Archetype T

$$T\colon P_4 \mapsto P_5, \quad T\left(p(x)\right) = (x - 2)p(x)$$

Archetype U

$$T\colon M_{23} \mapsto \mathbb{C}^4, \quad T\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} a+2b+12c-3d+e+6f \\ 2a-b-c+d-11f \\ a+b+7c+2d+e-3f \\ a+2b+12c+5e-5f \end{bmatrix}$$

Archetype V

$$T\colon P_3 \mapsto M_{22}, \quad T\left(a+bx+cx^2+dx^3\right) = \begin{bmatrix} a+b & a-2c \\ d & b-d \end{bmatrix}$$

Archetype W

$$T\colon P_2 \mapsto P_2, \quad T\left(a+bx+cx^2\right) = (19a+6b-4c)+(-24a-7b+4c)+(36a+12b-9c)$$

Archetype X

$$T\colon M_{22} \mapsto M_{22}, \quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} -2a+15b+3c+27d & 10b+6c+18d \\ a-5b-9d & -a-4b-5c-8d \end{bmatrix}$$